The pseudoinverse method for (approximate) integer division approximates multiplication by x with multiplication by $\overline{x}$, where $\overline{x} = x + \Delta x$, and $\Delta x \geq 0$. We wish to establish a bound on the smallest value of k such that $\lfloor \overline{x}k \rfloor \neq \lfloor xk \rfloor$. Since $\overline{x} \geq x$, (and we wish to minimise k), the latter is equivalent to $\lfloor \overline{x}k \rfloor = \lfloor xk \rfloor + 1$. Let $k = n/x + k'$, where $n, k' \in \mathbb{N}$ and $k' < 1/x$.

$$\begin{array}{lrcl} & \lfloor \overline{x}k \rfloor & = & \lfloor xk \rfloor + 1 \\ \Leftrightarrow & \lfloor xk + \Delta xk \rfloor & = & \lfloor xk \rfloor + 1 \\ \Leftrightarrow & \lfloor n + xk' + \frac{n\Delta x}{x} + k'\Delta x \rfloor & = & n + 1 \\ \Leftrightarrow & \lfloor xk' + \frac{n\Delta x}{x} + k'\Delta x \rfloor & = & 1 \\ \Rightarrow & xk' + \frac{n\Delta x}{x} + k'\Delta x & \geq & 1 \\ \Leftrightarrow & n & \geq & (1 - k'\overline{x}) \frac{x}{\Delta x} \end{array}$$

n is minimised by bringing $k'\overline{x}$ as close to 1 as possible; let $k' = \left\lceil \frac{1}{x} \right\rceil - 1$.

$$\begin{array}{lrcl} & n & \geq & (1 - k'\overline{x}) \frac{x}{\Delta x} \\ \Leftrightarrow & n & \geq & [1 - k'(x + \Delta x)] \frac{x}{\Delta x} \\ \Leftrightarrow & n & \geq & \left[1 - x\frac{1}{x} - \Delta x\frac{1}{x} + \overline{x}\right] \frac{x}{\Delta x} \\ \Leftrightarrow & n & \geq & \frac{x\overline{x}}{\Delta x} - 1 \end{array}$$

So, for example, with $x = 1/7$, approximated with $\overline{x} = 9363/2^{16} \approx x + 1.0899135 \cdot 10^{-5}$, we find $n \geq 1872$, and $k = n/x + \lceil 1/x \rceil - 1 = 13110$. And, indeed, $\lfloor \overline{x}k \rfloor = 1873 \neq \lfloor xk \rfloor = 1872$. (But with $n - 1$ instead of n, $\overline{x}$ still yields the same result)